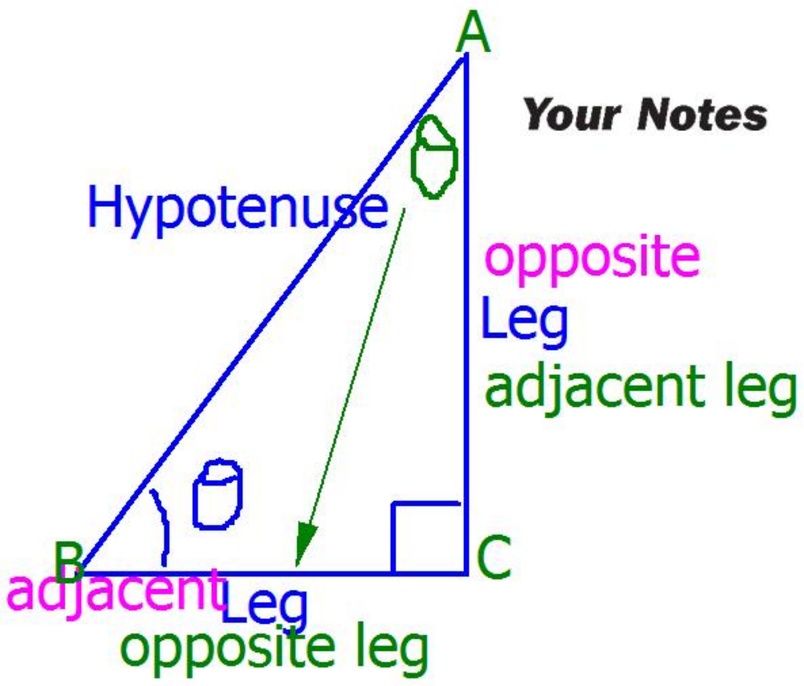
13.1 Use Trigonometry with Right Triangles

Goal • Use trigonometric functions to find lengths.



Your Notes

VOCABULARY

Sine, cosine, tangent, cosecant, secant, cotangent

RATIOS of any three sides of a right triangle

RIGHT TRIANGLE DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows:

$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite leg}}$$ reciprocal of sine

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$ reciprocal of cosin

$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite leg}}$$ reciprocal of tangent

The abbreviations *opp*, *adj*, and *hyp* are often used to represent the side lengths of the right triangle. Note that the ratios in the second column are reciprocals of the ratios in the first column:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Your Notes

Pythagorean Triples

3,4,5 5,12,13
6,8,10 10,24,26
9,12,15

7,24,25 8,15,17

$\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

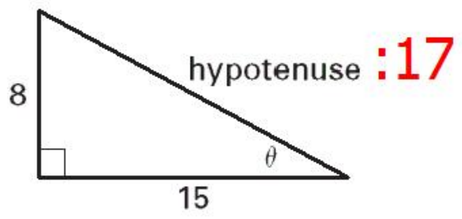
$\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}\ \frac{\sqrt{3}}{3}$

Example 1 *Evaluate trigonometric functions*

Evaluate the six trigonometric functions of the angle θ.

From the Pythagorean theorem, the length of the hypotenuse is

$\sqrt{8^2 + 15^2} = \sqrt{289} = 17$.



$\sin\theta = \frac{\text{Opp}}{\text{hyp}} = \frac{8}{17}$ $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$

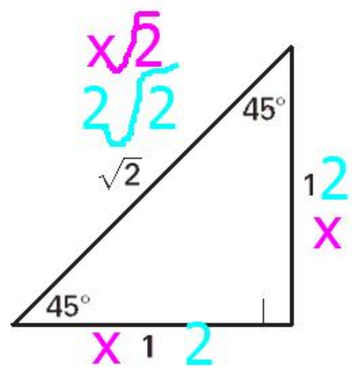
$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$

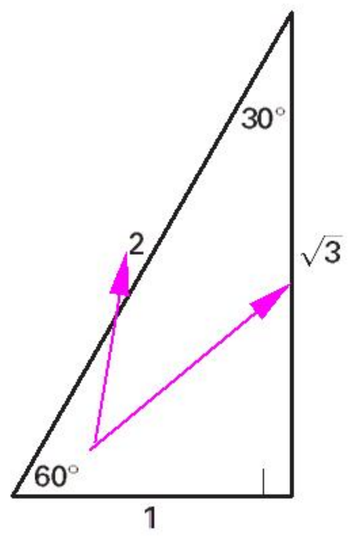
TRIGONOMETRIC VALUES FOR SPECIAL ANGLES

The table below gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°. You can obtain these values from the triangles shown.

θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$



θ	csc θ	sec θ	cot θ
30°	2	$\frac{2\sqrt{3}}{3}$	_____
45°	$\sqrt{2}$	_____	1
60°	_____	2	$\frac{\sqrt{3}}{3}$



Your Notes

REMEMBER THIS!

sine = opp/hyp
cosine = adj/hyp
tangent = opp/adj

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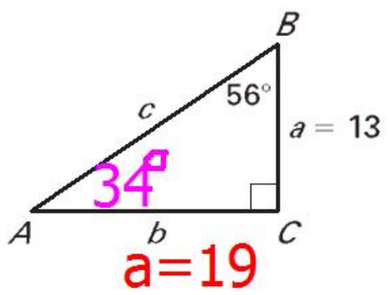
Example 2 *Use a calculator to solve a right triangle*

Solve $\triangle ABC$.



a=19

Solution

A and B are complementary angles, so $A = 90° - \underline{56} = \underline{34}$.

$$\tan 56° = \frac{\text{opp}}{\text{adj}} \qquad \sec 56° = \frac{\text{hyp}}{\text{adj}}$$

$$\tan 56° = \frac{b}{13} \qquad \sec 56° = \frac{c}{a}$$

$13*\tan 56 = b$ $\qquad a\left(\frac{1}{\cos 56}\right) = c$

$19 \approx b$ $\qquad 23 \approx c$

So, $A = 13$, $b \approx 19$, and $c \approx 23$.

$\sqrt{13^2+19^2}$

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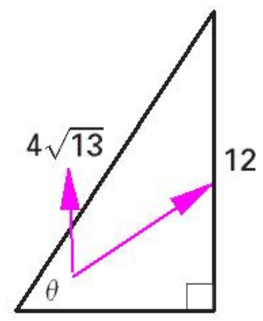
Checkpoint **Complete the following exercises.**

1. Evaluate the six trigonometric functions of the angle θ.

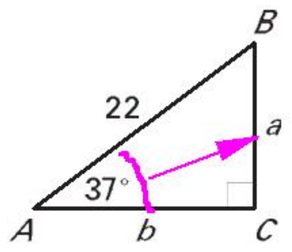


$\sin\theta = 12/4\sqrt{13}$

$\theta = \text{SIN}^{-1}(12 / 4/\sqrt{13})$

$= 56°$

2. Solve $\triangle ABC$.



$\sin 37° = \frac{\text{opp}}{\text{hyp}} = \frac{a}{22}$

$a = 22*\sin 37° = 13.24$ units

$b = \sqrt{c^2 - a^2} = 17.56$ units

Your Notes

Example 3 ***Use an angle of elevation***

Building Height You are measuring the height of your school building. You stand 25 feet from the base of the school. The angle of elevation from a point on the ground to the top of the school is 62°. Estimate the height of the school to the nearest foot.

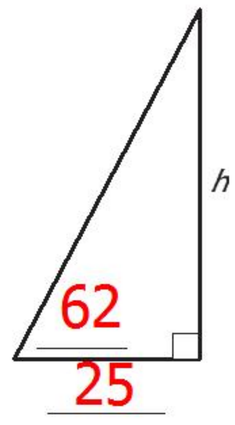
Solution

1. **Draw** a diagram that represents the situation.
2. **Write** and solve an equation to find the height h.

$$\tan 62 = \frac{h}{25}$$

$$25(\tan 62°) = h$$

$$47 \approx h$$



The height of the school is about 47 feet.

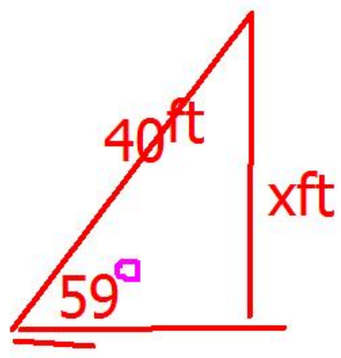
✔ ***Checkpoint*** **Complete the following exercise.**

3. A kite makes an angle of 59° with the ground. If the string on the kite is 40 feet, how far above the ground is the kite itself? Round to the nearest foot.

$$\sin 59° = \frac{X}{40}$$

$$40 * \sin 59 = X$$

$$X = 34.28 \text{ feet}$$



always check the reasonableness of your answers
34 feet is shorter which is reasonable becaus
40 ft is the hypotenuse, the longest side

Homework